Networks of Evolutionary Processors: Words and Pictures

Group of Mathematical Modelling and Bioinspired Algorithms

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Summary

1. Preliminaries
2. Evolutionary Pictures Operations
3. Circular Permutation Rules
4. Filters Predicates
5. Picture Processors
6. Accepting Networks of Picture Processors
7. Problems

1. Preliminaries

POLITÉCNICA

- **Goal**: To apply distributed computing methods to networks of sub-microprocessor devices, e.g., biological cellular networks or networks of nano-devices.
- **Question**: do tiny bio/nano nodes "compute" and/or "communicate" essentially the same as a computer?
- **Our attempt**: Although the computation and communication capabilities of each individual device in the new model are, by design, much weaker than those of a computer, we show that some of the most important and extensively studied distributed computing problems can still be solved efficiently.
- Theorists: YES, WE CAN!
- Engineers: ?
- Biologists: ?

1. Preliminaries

- V is an alphabet

$$V^* = \{\omega \mid \omega = \omega_1\omega_2 \ldots \omega_p \,,\ \omega_i \in V \ \wedge p \ \in \mathbb{N}\}$$

- ω is a word over V and V* is the set of words over V
- The alphabet of ω is defined by $alph(\omega) = \{\, a \in V \mid a \ \in \omega\}$
- $L \ \subseteq V^*$ is a language over V

1. Preliminaries

- A picture π over an alphabet *V* is a two-dimensional array of elements from *V*.

$$\pi = (p_i^j)_{i\in\mathbb{N}}^{j\in\mathbb{N}} \wedge p_i^j \in V$$

$$V_*^* = \{\pi |\ \pi\ is\ a\ picture\ over\ V\}$$

$\overline{\underline{\pi}}$ is the number of rows of π

$|\pi|$ is the number of columns of π

size (π) = ($\overline{\underline{\pi}}$, $|\pi|$)

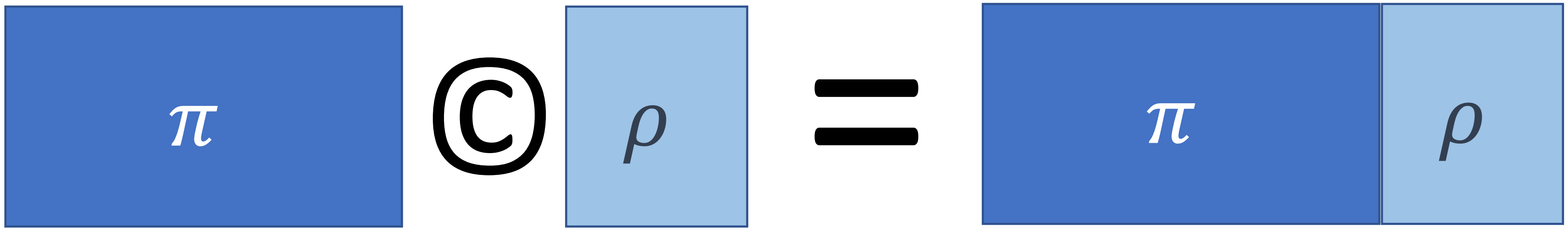
2. Evolutionary Pictures Operations

- Row and column concatenation operations

$$Let\ \pi\ and\ \rho \in V_*^*\ two\ pictures\ over\ V$$
$$size(\pi) = (m, n), size\ (\rho) = (m', n')$$

- π © ρ is the column concatenation of π and ρ and it is defined only if

m = m'



2. Evolutionary Pictures Operations

- Row and column concatenation operations

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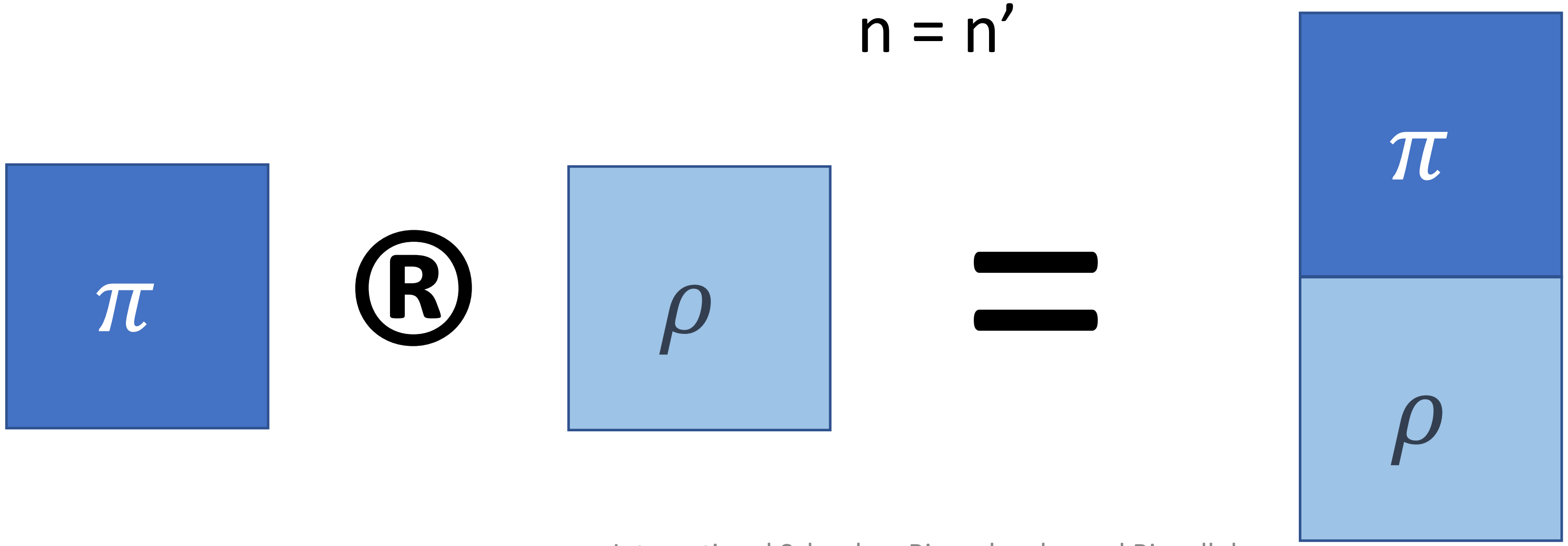
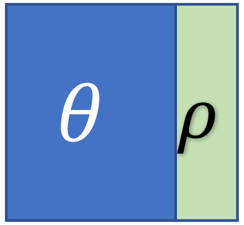


2. Evolutionary Pictures Operations

- Column right-quotient of π with ρ

$$\pi / \leftarrow \rho = \theta \Leftrightarrow \pi = \theta \copyright \rho$$



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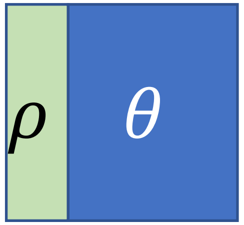
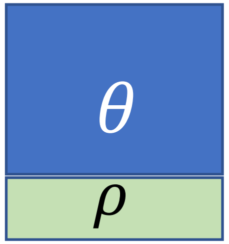


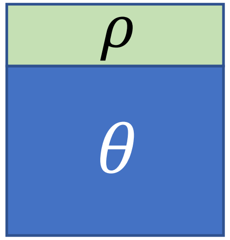
- Row down – quotient of π with ρ

$$\pi / \downarrow \rho = \theta \Leftrightarrow \pi = \theta \circledR \rho$$



- Row up – quotient of π with ρ

$$\pi / \uparrow \rho = \theta \Leftrightarrow \pi = \rho \circledR \theta$$



2. Evolutionary Pictures Operations

- Evolutionary rules

$$\sigma \equiv a \rightarrow b, \text{ is a:}$$

- substitution rule iff $a \neq \varepsilon \wedge b \neq \varepsilon$
- deletion rule iff $a \neq \varepsilon \wedge b = \varepsilon$
- insertion rule iff $a = \varepsilon \wedge b \neq \varepsilon$

2. Evolutionary Pictures Operations

- Definitions:

$$Sub_V = \{\sigma \mid \sigma \text{ is a substitution rule}\}$$

$$Del_V = \{\sigma \mid \sigma \text{ is a deletion rule}\}$$

2. Evolutionary Pictures Operations

- Actions for evolutionary rules when are applied on pictures

Given a rule σ an a picture π $\in V_m^n$ we defined the following actions:

$$\sigma^{\leftarrow}(\pi)$$
$$\sigma^{\rightarrow}(\pi)$$
$$\sigma^{\uparrow}(\pi)$$
$$\sigma^{\downarrow}(\pi)$$

$$\sigma^{|}(\pi)$$
$$\sigma^{-}(\pi)$$
$$\sigma^{+}(\pi)$$

2. Evolutionary Pictures Operations

- If $\sigma \equiv a \rightarrow b \in Sub_V$ *then*
  - *If the first column of contains an occurrence of a, then* $\sigma^{\leftarrow}(\pi)$ *is the set of all pictures* π' *such that the following conditions are satisfied:*
    i. $\exists\, 1 \leq i \leq m, \pi(i,1) = a \wedge \pi'(i,1) = b,$
    ii. $\pi(k,l) = \pi'(k,l)\ for\ all\ (k,l) \in [m]x[n] - \{(i,1)\}.$
  - *If this column does not contain any occurrence of a, then* $\sigma^{\leftarrow}(\pi)$ *=*$\{\pi\}$

2. Evolutionary Pictures Operations

$$\sigma^{\leftarrow}\left(\begin{bmatrix} a & b & b & c \\ c & b & b & a \\ c & c & b & c \\ a & b & c & b \end{bmatrix}\right) = \left\{\begin{bmatrix} b & b & b & c \\ c & b & b & a \\ c & c & b & c \\ a & b & c & b \end{bmatrix}, \begin{bmatrix} a & b & b & c \\ c & b & b & a \\ c & c & b & c \\ b & b & c & b \end{bmatrix}\right\}$$

$$\sigma^{\leftarrow}\left(\begin{bmatrix} c & b & b & c \\ c & b & b & a \\ c & c & b & c \\ b & b & c & b \end{bmatrix}\right) = \left\{\begin{bmatrix} c & b & b & c \\ c & b & b & a \\ c & c & b & c \\ b & b & c & b \end{bmatrix}\right\}$$

2. Evolutionary Pictures Operations

- In an analogous way can be defined

$$\sigma^{\rightarrow}(\pi), \sigma^{\uparrow}(\pi), \sigma^{\downarrow}(\pi), \sigma^{+}(\pi)$$

as the set of all pictures obtained by applying σ to the right most column, to the first row, to the las row, and to any column/row of π.

2. Evolutionary Pictures Operations

$$\sigma^{\rightarrow}\left(\begin{bmatrix} a & b & b & c \\ c & b & b & a \\ c & c & b & c \\ a & b & c & b \end{bmatrix}\right) = \left\{\begin{bmatrix} a & b & b & c \\ c & b & b & b \\ c & c & b & c \\ a & b & c & b \end{bmatrix}\right\}$$

$$\sigma^{\uparrow}\left(\begin{bmatrix} a & b & b & c \\ c & b & b & a \\ c & c & b & c \\ a & b & c & b \end{bmatrix}\right) = \left\{\begin{bmatrix} b & b & b & c \\ c & b & b & a \\ c & c & b & c \\ a & b & c & b \end{bmatrix}\right\}$$

2. Evolutionary Pictures Operations

$$\sigma^{\downarrow}\left(\begin{bmatrix} a & b & b & c \\ c & b & b & a \\ c & c & b & c \\ a & b & c & b \end{bmatrix}\right) = \left\{\begin{bmatrix} a & b & b & c \\ c & b & b & a \\ c & c & b & c \\ b & b & c & b \end{bmatrix}\right\}$$

$$\sigma^{+}\left(\begin{bmatrix} a & b & b & c \\ c & b & b & a \\ c & c & b & c \\ a & b & c & b \end{bmatrix}\right) = \left\{\begin{bmatrix} b & b & b & c \\ c & b & b & a \\ c & c & b & c \\ a & b & c & b \end{bmatrix}\begin{bmatrix} a & b & b & c \\ c & b & b & b \\ c & c & b & c \\ a & b & c & b \end{bmatrix}\begin{bmatrix} a & b & b & c \\ c & b & b & a \\ c & c & b & c \\ b & b & c & b \end{bmatrix}\right\}$$

2. Evolutionary Pictures Operations

- If $\sigma \equiv a \to \varepsilon \in Del_V$ then

$$\sigma^{\leftarrow}(\pi) = \begin{cases} \pi / \leftarrow \rho \\ \pi, otherwise \end{cases}$$

Where ρ is the leftmost column of π, and ρ contains at least one occurrence of ‘a’

$$\sigma^{\leftarrow}\left(\begin{bmatrix} a & b & b & c \\ c & b & b & a \\ c & c & b & c \\ a & b & c & b \end{bmatrix}\right) = \begin{bmatrix} b & b & c \\ b & b & a \\ c & b & c \\ b & c & b \end{bmatrix}$$

2. Evolutionary Pictures Operations

- Analogously are defined

$$\sigma^{\rightarrow}(\pi), \sigma^{\uparrow}(\pi), \sigma^{\downarrow}(\pi)$$

2. Evolutionary Pictures Operations

- Furthermore $\sigma^{|}(\pi)$ $(\sigma^{-}(\pi))$ is the set obtained from π by deleting an arbitrary column (row) of π containing the symbol ‘a’, then each column (row) is removed from different copies of π.
- If π does not contain a copy of ‘a’ then

$$\sigma^{|}(\pi) = \sigma^{-}(\pi) = \{\pi\}$$

2. Evolutionary Pictures Operations

$$\sigma^{|}\left(\begin{bmatrix} a & b & b & a \\ c & b & b & a \\ c & c & b & c \\ a & b & c & b \end{bmatrix}\right) = \left\{\begin{bmatrix} b & b & a \\ b & b & a \\ c & b & c \\ b & c & b \end{bmatrix}, \begin{bmatrix} a & b & b \\ c & b & b \\ c & c & b \\ a & b & c \end{bmatrix}\right\}$$

2. Evolutionary Pictures Operations

- Definitions:

i. For every rule σ, symbol $\alpha = \{\leftarrow, \rightarrow, \uparrow, \downarrow, |, -, +\}$ and $L \subseteq V_*^{*}$; we define the α-action of σ on *L* by

$$\sigma^{\alpha}(L) = \bigcup_{\pi \in L} \sigma^{\alpha}(\pi)$$

Note: + is defined only for substitution rules M, while | and – are defined only for deletion rules.

2. Evolutionary Pictures Operations

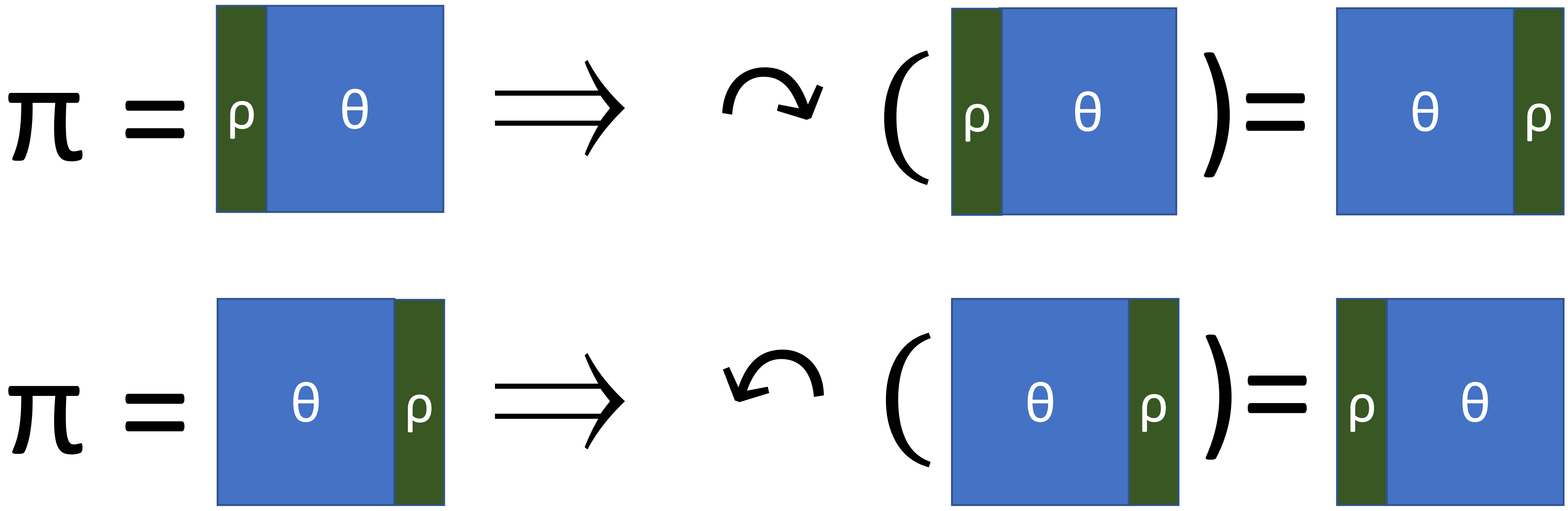
Definitions:

ii. Given a finite set of rules *M*, we define the α-action of *M* on the picture π and the language *L by*

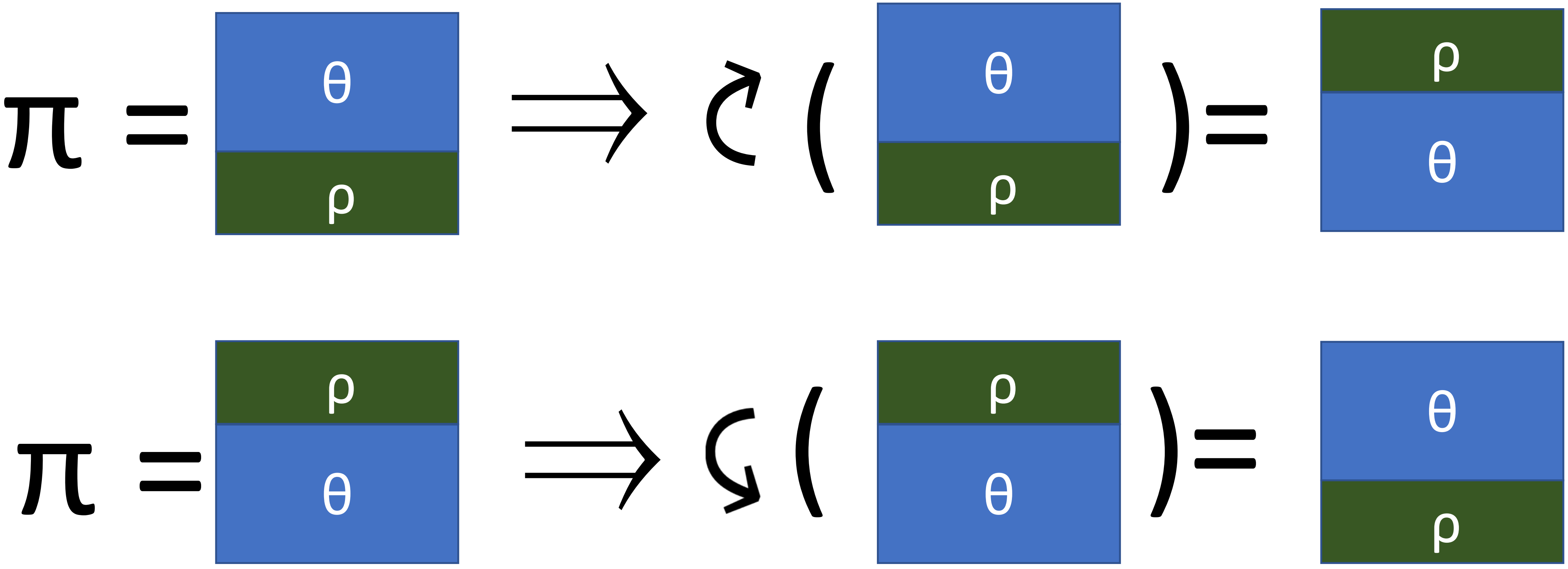
$$M^{\alpha}(\pi) = \bigcup_{\sigma \in M} \sigma^{\alpha}(\pi) \text{ and}$$

$$M^{\alpha}(L) = \bigcup_{\pi \in L} M^{\alpha}(\pi)$$

3. Circular Permutation Rules



3. Circular Permutation Rules



3. Circular Permutation Rules

- Definitions:

  For every $\circ \in \{\curvearrowleft, \curvearrowright, ⤹, ⤸\}$ and $L \subseteq V_*^*$,
  we define

$$\circ(L) = \{\circ(\pi) | \pi \in L\}$$

4. Filters Predicates

- As usual, we define two different policies for defining filters, strong and week defined by the following two predicates.
- Consider $P, F \subseteq V$ the set of permitting and forbidding symbols respectively

$$rc_s(\pi; P, F) \equiv P \subseteq alph(\pi) \land F \cap alph(\pi) = \emptyset$$
$$rc_w(\pi; P, F) \equiv P \cap alph(\pi) \neq \emptyset \land F \cap alph(\pi) = \emptyset$$

4. Filters Predicates

- For every picture language $L \subseteq V_*^*$ and $\beta \in \{s, w\}$ we define:

$$rc_\beta(L; P, F) = \{\pi \in L | rc_\beta(\pi; P, F) = true\}$$

5. Picture Processors

- Evolutionary Picture Processor (EPP)

An evolutionary picture Processor (EPP) over *V* is a 5-tuple

(M,PI,FI,PO,FO)

where:

- $M \subseteq Sub_V$ *or* $M \subseteq Del_V$ *(specialization)*
- $PI, FI \subseteq V$ *(permitting / forbidding input symbols)*
- $PO, FO \subseteq V$ *(permitting / forbidding outpt symbols)*

5. Picture Processors

- Circularly Permuted Picture Processor (CPPP)

A CPPP over *V* is a 5-tuple

$$(M, PI, PO, FI, FO)$$

where:

- ∘ ∈ {↶, ↷, ⤾, ⤿}
- The rest of parameters are identical to those defined for evolutionary processors.

6. Accepting Networks of Picture Processors (ANPP)

- An ANPP is a 9-tuple

$$\Gamma=(V, U, G, N, \alpha, \beta, \underline{In}, \underline{Halt}, \underline{Accept})$$

where:

- *V*, *U* are the input and network alphabet respectively $V \subseteq U$,
- $G = (X_G, E_G)$ is an undirected graph without loops,
- *N* is a mapping which associates each node $x \in X_G$ the picture processor

$$N(x) = (M_x, PI_x, PO_x, FI_x, FO_x)$$

6. Accepting Networks of Picture Processors (ANPP)

- $\alpha: X_G \rightarrow \{\rightarrow, \leftarrow, \uparrow, \downarrow, |, -, *\}$ gives the action mode of the rules of node *x* on the pictures existing in that node. If *x* is a CPPP then $\alpha(x) = *$
- $\beta: X_G \rightarrow \{s, w\}$ defines the type of input and output filters

$$\text{Input filter: } \rho_x(.) = rc_{\beta(x)}(.; PI_x, FI_x)$$

$$\text{Output filter: } \tau_x(.) = rc_{\beta(x)}(.; PO_x, FO_x)$$

- *In*, *Halt*, *Accept* $\in X_G$ are the input node, the halting node and the accepting node of *Γ* respectively.

6. Accepting Networks of Picture Processors (ANPP)

- <u>Definitions:</u>

1. We say that $Card(X_G)$ is the size of Γ.
2. A configuration of an ANPP Γ is a mapping

$$C: X_G \to 2^{U_*^*}$$

which associates a finite set of pictures with every node in the graph.

6. Accepting Networks of Picture Processors (ANPP)

- Given a picture $\pi \in V_*^*$, the initial configuration of Γ on π is defined by

$$C_0^{(\pi)}(\underline{In}) = \{\pi\}$$

$$C_0^{(\pi)}(x) = \emptyset \; for \; all \; x \in X_G \backslash \{\underline{In}\}$$

A configuration can change via a processing step or a communication step

6. Accepting Networks of Picture Processors (ANPP)

- Processing step

$$C \Rightarrow C' \; iff$$

$$C'(x) = M_x^{\alpha(x)}\big(C(x)\big) \forall x \in X_G \backslash \{\underline{In}\}$$

6. Accepting Networks of Picture Processors (ANPP)

- Communication step

$$C \vdash C' iff$$

$$C'(x) = (C(x) \backslash \tau_x(C(x)) \cup \bigcup_{\{x,y\} \in E_G} \left(\tau_y(C(y)) \cap \rho_x(C(y))\right), \forall x \in X_G$$

6. Accepting Networks of Picture Processors (ANPP)

- <u>Definition:</u>

Let Γ be an ANPP; the configuration of Γ on an input picture $\pi \in V_*^*$ is a sequence of configurations

$$C_0^{(\pi)} \Longrightarrow C_1^{(\pi)} \vdash C_2^{(\pi)} \Longrightarrow \; \dots C_n^{(\pi)}$$

where $C_0^{(\pi)}$ is the initial configuration of Γ on π

6. Accepting Networks of Picture Processors (ANPP)

- A computation halts if there exits a configuration such that the set of pictures existing in the halting node is non-empty.
- The picture language decided by Γ is:

$$L(\Gamma) = \{\pi \in V_*^{\,*} | the\ computation\ of\ \Gamma\ on\ \pi\ halts\ with\ a\ non-empty\ accepting\ node\}$$

7. Problems

- 2-Dimensional pattern matching

Given a picture π one can decide whether π is a subpicture of a given picture

SOLVED

What about 2-disjoint or overlapping occurrences?

7. Problems

- **Th. [BBLM – TPNC 2014]**

1. *Let π be a picture of size (k; n) for some $1 \leq k \leq 3$ and $n \geq 1$. The language $\{\pi\}$ can be accepted by an ANEPP.*

2. *Given a finite set F of patterns of size (k; l) and (l; k) for all $1 \leq k \leq 3$ and $l \geq 1$, the pattern matching problem with patterns from F can be solved by ANEPPs in O(n+m+kl+k) computational (processing and communication) steps.*

H. Bordihn, P. Bottoni, A. Labella and V. Mitrana: "Solving 2D-Pattern Matching with Networks of Picture Processors", Proceedings of Theory and Practice of Natural Computing - Third International Conference,
157--168, 2014.

7. Problems

- Let F be a finite set of pictures; the picture language F_*^* is the minimal set of pictures such that:

i. $F \subseteq F_*^*$,

ii. If $\pi, \rho \in F_*^*, then\ \pi®\rho \in F_*^*\ and\ \pi©\rho \in F_*^*$ provided that both of them exist

For a given picture π, one can decide whether $\pi \in F_*^*$?

- If all picture of *F* are of the same size, THE PROBLEM IS SOLVED

7. Problems

- **Th. [BBLM – Soft Computing 2016]**
- *Let (k; l) be two positive integers and F be a finite set of pictures of size (k; l). The language* F_*^* *can be decided by ANEPPs in O(n + m + kl) computational (processing and communication) steps.*

H. Bordihn, P. Bottoni, A. Labella, V. Mitrana: "Networks of picture processors as problem solvers", Soft Computing, 2016.

7. Problems

- A picture π is row/column scattered in a picture ϑ if ϑ contains a subpicture ρ such that all the row/column of π appears as row/column of ρ in the same order.

For a given pattern π and an arbitrary picture ϑ one can decide whether π is row/column scattered in ϑ

6. Problems

- The Hamming distance between 2 pictures of the same size is the number of positions at which the corresponding symbols are different.

For a given pattern π, a positive integer k, and an arbitrary picture ϑ one can decide whether ϑ contains a subpicture that is at the Hamming distance of at most k from π?

Thank you very much