P Systems with fuzzy ingredients

Definitions and applications

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Research Group on Natural Computing

Universidad de Sevilla

Motivation

Membrane computing models inspired by structure and functioning of cells. Uncertainty is an inherent property of living systems.

Reasons to add uncertainty ingredients to P systems:

- Ability to deal with imprecise information.
- Semantics closer to behaviour of real cells.
- Real biological processes modellization.

Two approaches:

- Probabilistic ingredients.
- Fuzzy ingredients.

Fuzzy sets

Fuzzy predicate:

- Admit different degrees of truth.
  - *John is young.*
  - *Temperature is very high.*
- Represented by fuzzy sets.

Given a universe E:

- Classic or crisp set $A \subseteq E$: $\mu_A : E \to \{0, 1\}$
- Fuzzy set $A \subseteq E$: $\mu_A : E \to [0, 1]$

Membership functions

Finite universe $E = \{x_1, \dots, x_n\}$

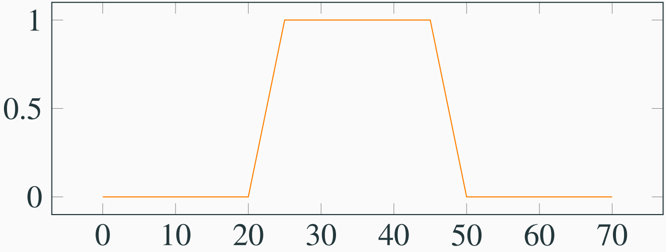
$$A = \mu_A(x_1)/x_1 + \cdots + \mu_A(x_n)/x_n$$

$$\textit{Ages} = \{20, 30, 40, 50\}$$
$$\textit{John is young} = .4/20 + 1/30 + .8/40 + .2/50$$

Membership functions

Continuous universe $E = \mathbb{R}$:

Trapezoidal (a, b, c, d): $\mu_A(x) = \max\left(\min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right)$

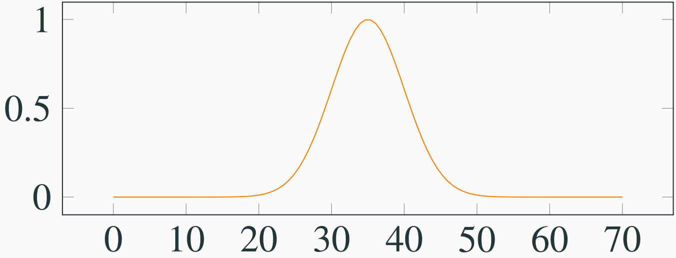


John is young = trapezoidal $(20, 25, 45, 50)$

Membership functions

Continuous universe $E = \mathbb{R}$:

Gaussian (μ, σ): $\mu_A(x) = e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$



John is young = gaussian $(35, 5)$

Fuzzy set operations

Intersection: $\mu_{A \cap B}(x) = T\big(\mu_A(x), \mu_B(x)\big)$

- $T : [0, 1]^2 \to [0, 1]$ is called a norm.
- Must verify

$$T(0, 0) = T(0, 1) = T(1, 0) = 0$$
$$T(1, 1) = 1$$

- Usually $T(x, y) = \min(x, y)$.

Fuzzy set operations

Union: $\mu_{A \cup B}(x) = S\big(\mu_A(x), \mu_B(x)\big)$

- $S\colon [0,1]^2 \to [0,1]$ is called a conorm.
- Must verify

$$S(0,1) = S(1,0) = S(1,1) = 1$$
$$S(0,0) = 0$$

- Usually $S(x,y) = \max(x,y)$.

Fuzzy set operations

Complement: $\mu_{E \setminus A}(x) = N\left(\mu_A(x)\right)$

- $N : [0, 1] \to [0, 1]$ is called a negation.
- Must verify

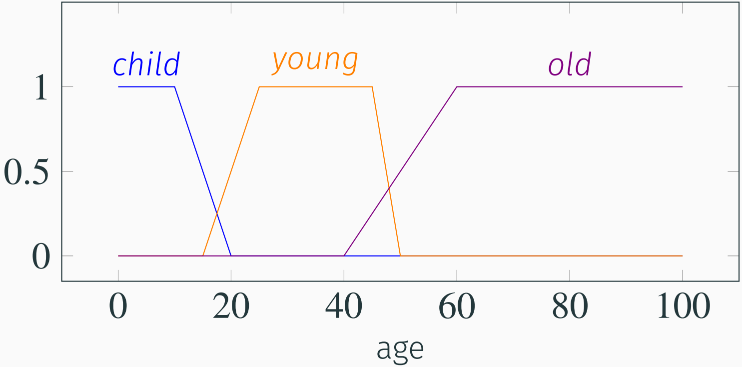
$$N(0) = 1$$
$$N(1) = 0$$

- Usually $N(x) = 1 - x$.

Linguistic variables

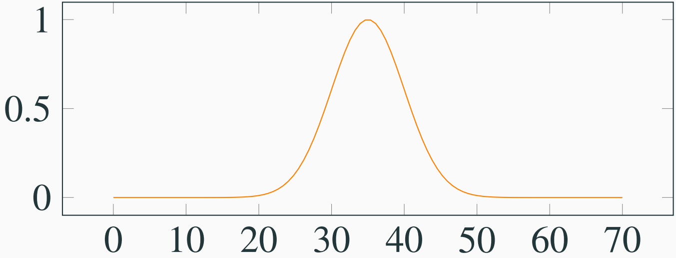
A linguistic variable is a set of fuzzy terms.

$Age = \{child, young, old\}$



Hedges

A hedge is an adjective or adverb that modify the truth value of a proposition.



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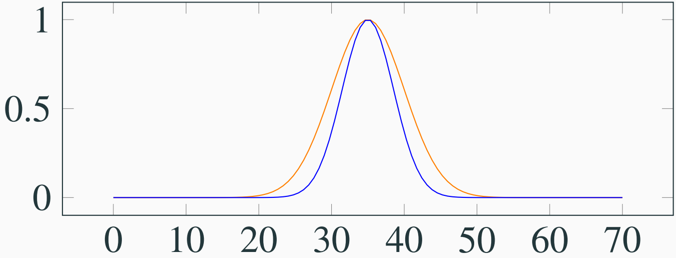
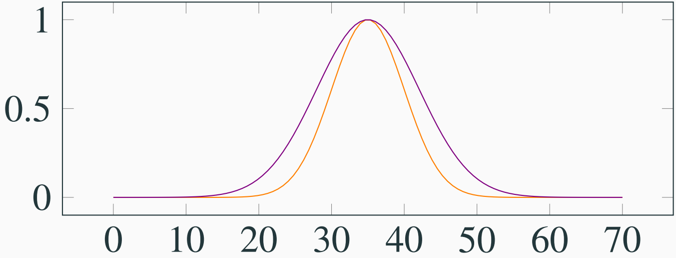


John is young = $\text{gaussian}(35, 5)$

John is very young = $\big(\text{gaussian}(35, 5)\big)^2$

Hedges

A hedge is an adjective or adverb that modify the truth value of a proposition.



John is young = $\text{gaussian}(35, 5)$

John is somewhat young = $\sqrt{\text{gaussian}(35, 5)}$

Fuzzy production systems

Production system is expert system with rules of the type

IF antecedent THEN consequent

In a fuzzy production system:

- Antecedent and consequent: logical combinations of fuzzy propositions involving linguistic terms.
- Logic truth of a logical combination: computed using a norm (conjunctions), a conorm (disjunctions) and a complement (negations).
- Uncertainty about the correctness of rules: confidence factors.

Example of rules

$\varphi_6 =$ Flow path of the combustor wears and tears
$\varphi_7 =$ Flow rate of the fuel in the combustor is too high
$\varphi_{18} =$ Compressor is in turbulence
$\varphi_{19} =$ Blade of compressor breaks down
$\varphi_{25} =$ Spray head of turbine is broken

IF φ_{19} THEN φ_{18} $(\tau = 1)$
IF φ_{25} AND NOT φ_7 THEN φ_6 $(\tau = 0.8)$

Types of rules

1. Simple fuzzy production rule:

$$\text{IF } \varphi \text{ THEN } \psi$$

2. Composite fuzzy conjunctive rules:

$$\text{IF } \varphi_1 \text{ AND } \cdots \text{ AND } \varphi_n \text{ THEN } \psi \qquad \text{(AND-rule)}$$
$$\text{IF } \varphi \text{ THEN } \psi_1 \text{ AND } \cdots \text{ AND } \psi_n$$

3. Composite fuzzy disjunctive rules:

$$\text{IF } \varphi_1 \text{ OR } \cdots \text{ OR } \varphi_n \text{ THEN } \psi \qquad \text{(OR-rule)}$$
$$\text{IF } \varphi \text{ THEN } \psi_1 \text{ OR } \cdots \text{ OR } \psi_n$$

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Inference

Given:

- A rule with confidence factor τ.
- A truth value α_i for each proposition φ_i in the antecedent.

Truth value derived for consequent ψ:

- AND-rules: $\min(\alpha_1, \ldots, \alpha_n) \cdot \tau$
- OR-rules: $\max(\alpha_1, \ldots, \alpha_n) \cdot \tau$

Inference

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- OR-rules: $\max(\alpha_1, \ldots, \alpha_n) \cdot \tau$

Final truth value for proposition ψ: $\max(\alpha_{r_1}, \ldots, \alpha_{r_m})$

- $r_1, \ldots, r_m$ all rules with consequent ψ.
- α_{r_i} truth value for ψ derived from r_i.

FRSN P Systems

A Fuzzy Reasoning Spiking Neural P System aims to represent and make inferences from a fuzzy production system.

FRSN P Systems

A Fuzzy Reasoning **Spiking Neural P System** aims to represent and make inferences from a fuzzy production system.

- Inspired by neuron inter-communications by means of short electrical impulses.
- Directed graph, the nodes representing neurons, the arcs representing synapses.
- Impulses described by multiplicity of an object a (spike) from a singleton alphabet.
- Neurons send spikes depending on their current number of spikes.

FRSN P Systems

A **Fuzzy Reasoning** Spiking Neural P System aims to represent and make inferences from a fuzzy production system.

- Proposition neurons: represent fuzzy propositions.
  - Input neurons: receive no spike.
  - Output neurons: send no spike.
- Rule neurons: represent production rules.
  - AND-type neurons.
  - OR-type neurons.
- Synapses connect proposition neurons with rule neurons or vice versa.

Formalization

FRSN P System of degree $q \geq 1$:

$$\left(A, \sigma_1, \ldots, \sigma_q, \mathit{syn}, \mathit{IN}, \mathit{OUT}, \mathit{PN}, \mathit{RN}_{\mathrm{AND}}, \mathit{RN}_{\mathrm{OR}}, \boldsymbol{P}, \boldsymbol{C}\right)$$

- $A = \{a\}$ singleton alphabet.
- (V, syn) directed graph:
    - $V = \mathit{PN} \amalg \mathit{RN}_{\mathrm{AND}} \amalg \mathit{RN}_{\mathrm{OR}} = \{\sigma_1, \ldots, \sigma_q\}$.
    - $\mathit{indegree}(\sigma) > 0 \vee \mathit{outdegree}(\sigma) > 0$, for all $\sigma \in V$.
    - $\left(\sigma \in \mathit{PN} \wedge \sigma' \in \mathit{RN}\right) \vee \left(\sigma \in \mathit{RN} \wedge \sigma' \in \mathit{PN}\right)$, for all $\left(\sigma, \sigma'\right) \in \mathit{syn}$.
    - A function $\gamma_{(\sigma,\sigma')}\colon [0,1] \to [0,1]$ associated with each arc $\left(\sigma, \sigma'\right) \in \mathit{syn}$.
- $\mathit{IN} = \{\sigma \in \mathit{PN} \mid \mathit{indegree}(\sigma) = 0\}$ nonempty.
- $\mathit{OUT} = \{\sigma \in \mathit{PN} \mid \mathit{outdegree}(\sigma) = 0\}$ nonempty.

Formalization

- $P = (p_1, \ldots, p_q), p_i \in [0, 1]$.
- $C = (c_1, \ldots, c_q), c_i \in [0, 1]$, $c_i > 0$ if $\sigma_i \in RN$.
- Each node $\sigma_i = (\alpha_i, c_i, F_i)$:
  - $\alpha_i \in [0, 1]$.
  - F_i the identity over $[0, 1]$ for $\sigma_i \in IN$.
  - If *indegree*$(\sigma_i) = n_i > 0$ then $F_i \colon [0, 1]^{n_i} \to [0, 1]$ and the value $F_i(x_1, \ldots, x_{n_i})$ is defined as follows:
    - $\max(\gamma_1(x_1), \ldots, \gamma_{n_i}(x_{n_i}))$, if $\sigma_i \in PN$.
    - $\max(\gamma_1(x_1), \ldots, \gamma_{n_i}(x_{n_i})) \cdot c_i$, if $\sigma_i \in RN_{\mathrm{OR}}$.
    - $\min(\gamma_1(x_1), \ldots, \gamma_{n_i}(x_{n_i})) \cdot c_i$, if $\sigma_i \in RN_{\mathrm{AND}}$.

    where γ_j function associated with j-th incoming arc of σ_i.

FRSN P System from fuzzy production system

Rules of the production system: $\{r_1, \ldots, r_{n_r}\}$
Propositions present in the rules: $\{\varphi_1, \ldots, \varphi_{n_p}\}$

Construct the following FRSN P System:

$$\left(A, \sigma_1, \ldots, \sigma_q, syn, IN, OUT, PN, RN_{\mathrm{AND}}, RN_{\mathrm{OR}}, \boldsymbol{P}, \boldsymbol{C}\right)$$

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One neuron for each proposition and each rule:

$$PN = \{\sigma_1, \ldots, \sigma_{n_p}\}$$
$$RN_{\mathrm{AND}} = \{\sigma_{n_p+\mathrm{i}} \mid r_i \text{ an AND-rule}\}$$
$$RN_{\mathrm{OR}} = \{\sigma_{n_p+\mathrm{i}} \mid r_i \text{ an OR-rule}\}$$

FRSN P System from fuzzy production system

Rules of the production system: $\{r_1, \ldots, r_{n_r}\}$
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Construct the following FRSN P System:

$$(A, \sigma_1, \ldots, \sigma_q, syn, IN, OUT, PN, RN_{\mathrm{AND}}, RN_{\mathrm{OR}}, \boldsymbol{P}, \boldsymbol{C})$$

For each rule $\boldsymbol{r}_i$:

- An arc $(\sigma_j, \sigma_{n_p+i})$ for each φ_j antecedent of $\boldsymbol{r}_i$.
- An arc $(\sigma_{n_p+i}, \sigma_j)$ for each φ_j consequent of $\boldsymbol{r}_i$.
- $\gamma(x) = 1 - x$ if φ_j is negated in the rule, $\gamma(x) = x$ otherwise.

FRSN P System from fuzzy production system

Rules of the production system: $\{r_1, \ldots, r_{n_r}\}$
Propositions present in the rules: $\{\varphi_1, \ldots, \varphi_{n_p}\}$

Construct the following FRSN P System:

$$\left(A, \sigma_1, \ldots, \sigma_q, syn, IN, OUT, PN, RN_{\text{AND}}, RN_{\text{OR}}, \boldsymbol{P}, \boldsymbol{C}\right)$$

$$P = \{p_1, \ldots, p_{n_p}, p_{n_p+1}, \ldots, p_{n_p+n_r}\}$$

- p_i initial truth value of proposition φ_i.
- $p_{n_p+i} = 0$.

FRSN P System from fuzzy production system

Rules of the production system: $\{r_1, \ldots, r_{n_r}\}$
Propositions present in the rules: $\{\varphi_1, \ldots, \varphi_{n_p}\}$

Construct the following FRSN P System:

$$\left(A, \sigma_1, \ldots, \sigma_q, \mathit{syn}, \mathit{IN}, \mathit{OUT}, \mathit{PN}, \mathit{RN}_{\mathrm{AND}}, \mathit{RN}_{\mathrm{OR}}, P, C\right)$$

$C = \{c_1, \ldots, c_{n_p}, c_{n_p+1}, \ldots, c_{n_p+n_r}\}$

- $c_i = p_i$ for $i \leq n_p$.
- c_{n_p+i} confidence factor for rule r_i.

Configurations

State $s(\sigma, t)$ of σ at instant t: $\left(\alpha_\sigma(t), f_\sigma(t), G_\sigma(t)\right)$

- $\alpha_\sigma(t) \in [0, 1]$ represents pulse value in neuron σ.
- $f_\sigma(t)$ represents if neuron σ ready or not to fire.
- $G_\sigma(t) \in [0, 1]$ represents pulse value neuron σ will send.

Configuration C_t of Π at instant t: $\left(s(\sigma_1, t), \ldots, s(\sigma_q, t)\right)$

Initial configuration:

$$s(\sigma_i, 0) = \begin{cases} \left(p_i, 1, p_i\right), & \text{if } \sigma_i \in IN \\ \left(p_i, 0, 0\right), & \text{if } \sigma_i \notin IN \end{cases}$$

Transition steps

For $\sigma_i \in IN$:

$$s(\sigma_i, t+1) = \begin{cases} \left(p_i, 1, p_i\right), & \text{if } t < t_D \\ \left(p_i, 0, 0\right), & \text{if } t \geq t_D \end{cases}$$

t_D maximum length of paths from *IN* to *OUT* neurons.

Transition steps

For $\sigma_i \notin IN$:

If $\sigma_i \in PN \cup RN_{\mathrm{OR}}$

$$\alpha_{\sigma_i}(t+1) = \max\Big(\gamma\big(G_\sigma(t)\big) \,\Big|\, \sigma \in \mathit{Presyn}\big(\sigma_i\big) \wedge f_\sigma(t) = 1\Big)$$

If $\sigma_i \in RN_{\mathrm{AND}}$

$$\alpha_{\sigma_i}(t+1) = \min\Big(\gamma\big(G_\sigma(t)\big) \,\Big|\, \sigma \in \mathit{Presyn}\big(\sigma_i\big) \wedge f_\sigma(t) = 1\Big)$$

$$\mathit{Presyn}\big(\sigma_i\big) = \Big\{\sigma \,\Big|\, \big(\sigma, \sigma_i\big) \in \mathit{syn}\Big\} = \Big\{\sigma_i^1, \ldots, \sigma_i^{m_i}\Big\}$$

Transition steps

For $\sigma_i \notin IN$:

$f_{\sigma_i}(t+1) = 1$ if and only if $\sigma_i \notin OUT$ and $f_\sigma(t) = 1$, for all $\sigma \in Presyn\left(\sigma_i\right)$

$$G_{\sigma_i}(t+1) = \begin{cases} 0, & \text{if } f_{\sigma_i}(t+1) = 0 \\ F_{\sigma_i}\left(G_{\sigma_i^1}(t), \ldots, G_{\sigma_i^{m_i}}(t)\right), & \text{if } f_{\sigma_i}(t+1) = 1 \end{cases}$$

$$Presyn\left(\sigma_i\right) = \left\{\sigma \mid \left(\sigma, \sigma_i\right) \in syn\right\} = \left\{\sigma_i^1, \ldots, \sigma_i^{m_i}\right\}$$

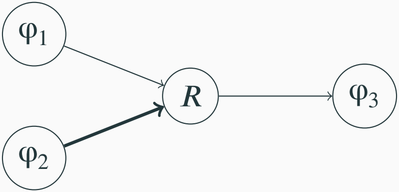
Halting configuration

A computation halts when it reaches a configuration C_t such that $f_\sigma(t) = 0$, for all neuron σ.

The result of a halting computation is encoded in the pulse values $\alpha_\sigma(t)$ of the output neurons $\sigma \in$ *OUT*.

Example of computation

Consider the rule $R =$ IF φ_1 AND NOT φ_2 THEN φ_3, with confidence factor τ. Suppose the truth value of φ_i is p_i.



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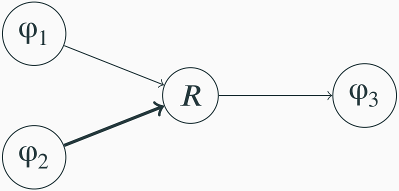
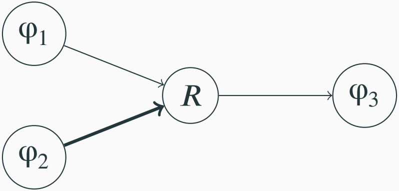


Computation:

$$C_0 = \left(p_1, 1, p_1\right), \left(p_2, 1, p_2\right), \left(p_3, 0, 0\right), (0, 0, 0)$$

Example of computation

Consider the rule $R =$ IF φ_1 AND NOT φ_2 THEN φ_3, with confidence factor τ. Suppose the truth value of φ_i is p_i.

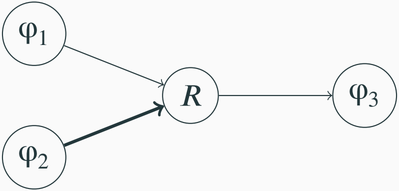


Computation:

$$\begin{aligned} C_1 = & (p_1, 1, p_1), (p_2, 1, p_2), (p_3, 0, 0), \\ & (\min(p_1, 1 - p_2), 1, \min(p_1, 1 - p_2) \cdot \tau) \end{aligned}$$

Example of computation

Consider the rule $R =$ IF φ_1 AND NOT φ_2 THEN φ_3, with confidence factor τ. Suppose the truth value of φ_i is p_i.

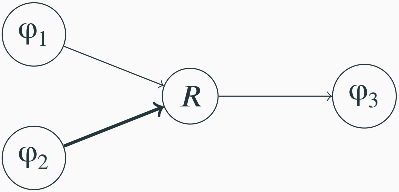


Computation:

$$C_2 = (p_1, 0, 0), (p_2, 0, 0), (\min(p_1, 1 - p_2) \cdot \tau, 0, 0),$$
$$(\min(p_1, 1 - p_2), 1, \min(p_1, 1 - p_2) \cdot \tau)$$

Example of computation

Consider the rule $R =$ IF φ_1 AND NOT φ_2 THEN φ_3, with confidence factor τ. Suppose the truth value of φ_i is p_i.



Computation:

$$C_3 = (p_1, 0, 0), (p_2, 0, 0), (\min(p_1, 1 - p_2) \cdot \tau, 0, 0), \\ (\min(p_1, 1 - p_2), 0, 0)$$

Application example: turbine fault diagnosis

Expert system with propositions

φ_1	Cross section area of turbine's path is too large (0.6)
φ_2	Efficiency of assembling unit is too low (1)
φ_3	Ventilation side of the guider's blade of turbine wears and tears
φ_4	Inlet gas temperature of turbine is too low (0)
φ_5	Pressurization ratio of the compressor is too low (0.2)
φ_6	Flow path of the combustor wears and tears
φ_7	Flow rate of the fuel in the combustor is too high (0)
φ_8	Higher pressure level's spray head of the turbine is broken
φ_9	Outlet gas temperature of turbine is too high (0.2)
φ_{10}	Efficiency of turbine is too low (0.9)
φ_{11}	Flow coefficient of turbine is too low (0)
φ_{12}	Blade of the turbine scales
φ_{13}	Power of assembling unit is too low (0.8)

Application example: turbine fault diagnosis

Expert system with propositions

φ_{14}	Blade of the turbine wears and tears
φ_{15}	Inlet gas temperature of turbine is too high (0.2)
φ_{16}	Blade of the turbine burns down
φ_{17}	Flow path of compressor wears and tears
φ_{18}	Compressor is in turbulence
φ_{19}	Blade of compressor breaks down (0)
φ_{20}	Conversion flow of the compressor is too low (0)
φ_{21}	Fuel consumption of assembling unit is too high (0.3)
φ_{22}	Inlet of compressor freezes
φ_{23}	Uniform entropy compression efficiency of compressor is too low (0)
φ_{24}	Compressor has a problem
φ_{25}	Spray head of turbine is broken

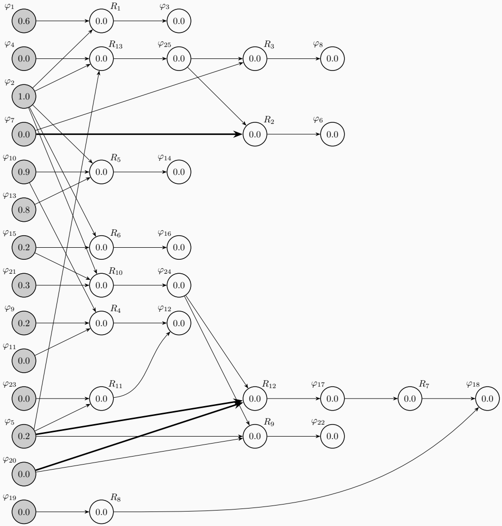
Application example: turbine fault diagnosis

and rules

R_1	IF φ_1 AND φ_2 THEN φ_3 (0.8)
R_2	IF φ_{25} AND NOT φ_7 THEN φ_6 (0.8)
R_3	IF φ_{25} AND φ_7 THEN φ_8 (0.8)
R_4	IF φ_9 AND φ_{10} AND φ_{11} THEN φ_{12} (0.8)
R_5	IF φ_{10} AND φ_{13} AND φ_2 THEN φ_{14} (0.8)
R_6	IF φ_2 AND φ_{15} THEN φ_{16} (0.8)
R_7	IF φ_{17} THEN φ_{18} (0.9)
R_8	IF φ_{19} THEN φ_{18} (1)
R_9	IF φ_{20} AND φ_5 AND φ_{24} THEN φ_{22} (0.8)
R_{10}	IF φ_2 AND φ_{21} AND φ_{15} THEN φ_{24} (1)
R_{11}	IF φ_5 AND φ_{23} THEN φ_{12} (0.9)
R_{12}	IF NOT φ_{20} AND NOT φ_5 AND φ_{24} THEN φ_{17} (0.8)
R_{13}	IF φ_4 AND φ_2 AND φ_5 THEN φ_{25} (0.8)

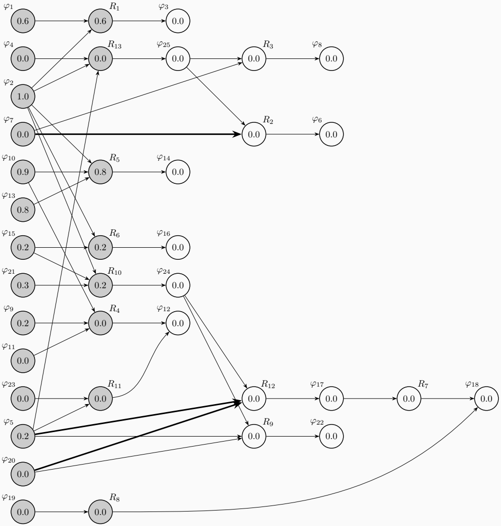
Application example: computation

Configuration at instant 0



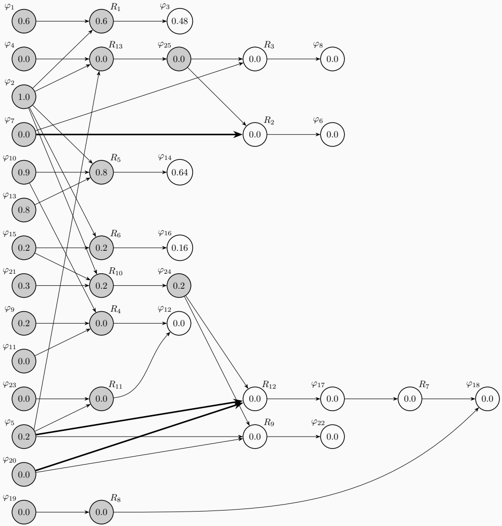
Application example: computation

Configuration at instant 1



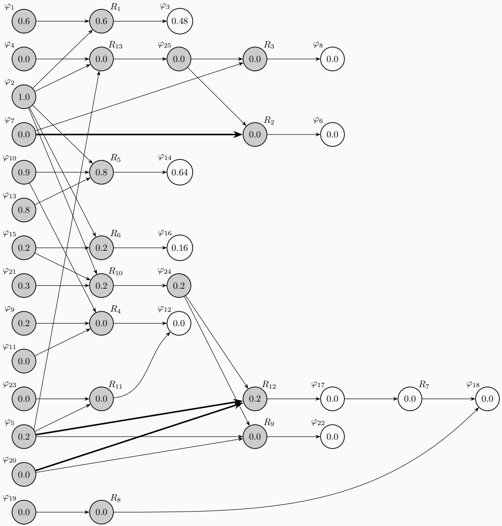
Application example: computation

Configuration at instant 2



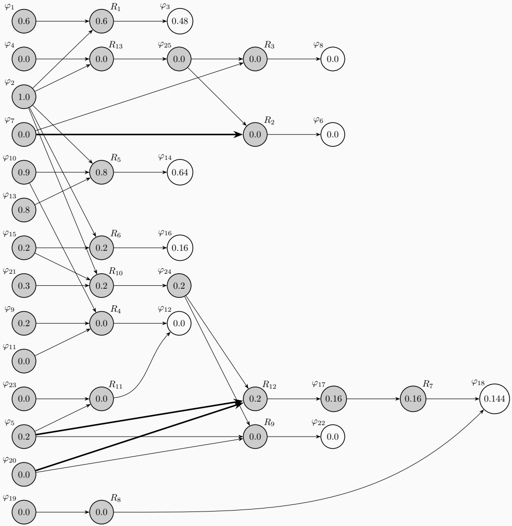
Application example: computation

Configuration at instant 3



Application example: computation

Configuration at instant 6



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