A theory of complexity for Membrane Computing

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Goal:

- Unconventional approaches/tools to attack the **P versus NP problem** are given by using **Membrane Computing**.

Computability versus Complexity

Computability (1931):

- Define the informal idea of mechanical/algorithmic problems resolution in a rigorous way.
- Which problems are computable in a (universal) model?

Complexity (1970):

- Provide bounds on the amount of resources necessary for every mechanical procedure (algorithm) that solves a given problem.
- Which (computable) problems are efficiently solvable?

The P versus NP problem

- The $\mathbf{P} \stackrel{?}{=} \mathbf{NP}$ question is one of the outstanding open problems in theoretical computer science.
- Whether or not finding solutions is harder than checking the correctness of solutions
- Whether or not discovering proofs is harder than verifying their correctness

- This is essentially the famous **P** versus **NP** problem

... the central problem of Computational Complexity theory

It is widely believed that it is harder

- finding (resp. proving) than checking (resp. verifying)
- to solve a problem than to check the correctness of a solution
- ... $\mathbf{P} \neq \mathbf{NP}$

Attacking the P versus NP problem

Classical approach (1970):

- **P** $=$ **NP**.
  - Find an **NP**-complete poblem such that it belongs to the class **P**.
- **P** $\neq$ **NP**.
  - Find an **NP**-complete poblem such that it does not belong to the class **P**.

Membrane Computing

- P systems provide nondeterministic models of computation.
- A computational complexity theory in Membrane Computing is proposed.
- Polynomial complexity classes associated with (cell–like and tissue–like) P systems are presented.
  - A notion of acceptance must be defined in the new (nondeterministic) framework.
    - We consider a definition of acceptance different than the classical one for nondeterministic Turing machines.

Recognizer Membrane Systems

- Cell-like P systems: $\Pi = (\Gamma, \Sigma, H, \mu, \mathcal{M}_1, \ldots, \mathcal{M}_q, \mathcal{R}, i_{in}, i_{out})$
- Tissue-like P systems: $\Pi = (\Gamma, \Sigma, \mathcal{E}, \mathcal{M}_1, \ldots, \mathcal{M}_q, \mathcal{R}, i_{in}, i_{out})$
  - The working alphabet contains two distinguished elements *yes* and *no*.
  - All computations halt.
  - For any computation of the system, either object *yes* or object *no* (but not both) must have been sent to the output region of the system, and only at the last step of the computation.

- Accepting/rejecting computations for recognizer P systems

Semantics

The rules of a membrane system are applied in a non-deterministic maximally parallel manner.

- Configuration.
  - *Initial configuration*.
  - *Halting configuration*.
- Transition step.
- Computation.
  - *Halting computation* (accepting or rejecting)

Polynomial time solvability by using membrane systems

- A decision problem X is *solvable in polynomial time* by a family of recognizer membrane systems $\mathbf{\Pi} = \{\Pi(n) : n \in \mathbf{N}\}$, iff:

  ★ The family $\mathbf{\Pi}$ is polynomially uniform by Turing machines, that is, there exists a DTM working in polynomial time which constructs the system $\Pi(n)$ from $n \in \mathbf{N}$.

  ★ There exists a pair (cod, s) of polynomial-time computable functions over I_X such that:

  (a) for each instance $u \in I_X$, $s(u)$ is a natural number and $cod(u)$ is an input multiset of the system $\Pi(s(u))$;

  (b) for each $n \in \mathbf{N}$, $s^{-1}(n)$ is a finite set;

  (c) the family $\mathbf{\Pi}$ is polynomially bounded with regard to (X, cod, s), that is, there exists a polynomial function p, such that for each $u \in I_X$ every computation of $\Pi(s(u))$ with input $cod(u)$ is halting and it performs at most $p(|u|)$ steps;

  (d) the family $\mathbf{\Pi}$ is sound with regard to (X, cod, s), that is, for each $u \in I_X$, if there exists an accepting computation of $\Pi(s(u))$ with input $cod(u)$, then $\theta_X(u) = 1$;

  (e) the family $\mathbf{\Pi}$ is complete with regard to (X, cod, s), that is, for each $u \in I_X$, if $\theta_X(u) = 1$, then every computation of $\Pi(s(u))$ with input $cod(u)$ is an accepting one.

- We denote it by $X \in \mathbf{PMC}_{\mathcal{R}}$

- $\mathbf{PMC}_{\mathcal{R}}$ is closed under complement and polynomial–time reductions.

Basic cell-like membrane systems

- $\Pi = (\Gamma, \Sigma, H, \mu, \mathcal{M}_1, \ldots, \mathcal{M}_q, \mathcal{R}, i_{in}, i_{out})$.
- *Basic transition* P systems:
  - $[\ u\ ]_h \rightarrow [\ v\ ]_h$ (evolution rules).
  - $[\ u\ ]_h \rightarrow v\ [\ \ ]_h$ and $u\ [\ \ ]_h \rightarrow [\ v\ ]_h$ (communication rules).
  - $[\ u\ ]_h \rightarrow v$ (dissolution rules).
- $\mathcal{T}$: class of recognizer basic transition P systems.

On efficiency of cell-like membrane systems

- **Proposition 1** (`Seville theorem`, 2004)
  Every DTM working in polynomial time can be simulated in polynomial time by a family of recognizer basic transition P systems.

- **Proposition 2** (`Milano theorem`, 2000)
  If a decision problem is solvable in polynomial time by a family of recognizer basic transition P systems, then there exists a DTM solving it in polynomial time.

- **Theorem:** $\mathbf{P} = \mathbf{PMC}_{\mathcal{T}}$ (Seville team, 2004).

  - **Corollary:** $\mathbf{P} \neq \mathbf{NP}$ if and only if every, or at least one, **NP**–complete problem is not in $\mathbf{PMC}_{\mathcal{T}}$.

P systems with active membranes

- Electrical charges.
- Type of rules:

  (a) $[\, a \to u \,]_h^{\alpha}$ *(object evolution rules)*.

  (b) $a\,[\ \ ]_h^{\alpha 1} \to [\, b \,]_h^{\alpha 2}$ *(send–in communication rules)*.

  (c) $[\, a \,]_h^{\alpha 1} \to [\ \ ]_h^{\alpha 2}\ b$ *(send–out communication rules)*.

  (d) $[\, a \,]_h^{\alpha} \to b$ *(dissolution rules)*.

  (e) $[\, a \,]_h^{\alpha 1} \to [\, b \,]_h^{\alpha 2}\ [\, c \,]_h^{\alpha 3}$ *(division rules for elementary membranes)*.

  (f) $[\,[\,]_{h_1}^{\alpha 1}\ [\,]_{h_2}^{\alpha 2}\,]_h^{\alpha} \to [\,[\,]_{h_1}^{\alpha 3}\,]_h^{\beta}\ [\,[\,]_{h_2}^{\alpha 4}\,]_h^{\gamma}$ *(division rules for non–elementary membranes)*.

- The sets $\mathcal{NAM}, \mathcal{AM}(+n)$ and $\mathcal{AM}(-n)$.

On efficiency of P systems with active membranes

- **Proposition 3:** A deterministic P system with active membranes but without membrane division can be simulated by a DTM with a polynomial slowdown.

  Theorem: $\mathbf{P} = \mathbf{PMC}_{\mathcal{NAM}}$.

- Efficient solutions to **NP**–complete problems in $\mathcal{AM}(-n)$:
  - $\mathbf{NP} \cup \mathbf{co\text{-}NP} \subseteq \mathbf{PMC}_{\mathcal{AM}(-n)}$ (Seville team 2003, A. Alhazov, C. Martín and L. Pan, 2004).

- A borderline between efficiency and non–efficiency: division rules in the framework of $\mathcal{AM}(-n)$.

- Bounds of the efficiency:
  - $\mathbf{PSPACE} \subseteq \mathbf{PMC}_{\mathcal{AM}(+n)}$ (A. Alhazov, C. Martín and L. Pan, 2003).
  - $\mathbf{PSPACE} \subseteq \mathbf{PMC}_{\mathcal{AM}(+n)} \subseteq \mathbf{EXP}$ (A.E. Porreca, G. Mauri and C. Zandron, 2006).

- **Conclusion:** the usual framework of $\mathcal{AM}$ for solving decision problems is too powerful from the complexity point of view.

Polarizationless P systems with active membranes

- $\Pi = (\Gamma, H, \mu, \mathcal{M}_1, \dots, \mathcal{M}_q, R, i_{in}, i_{out})$,

  (a) $[\, a \rightarrow u \,]_h$ (*object evolution rules*).

  (b) $a[\ ]_h \rightarrow [\, b \,]_h$ (*send–in communication rules*).

  (c) $[\, a \,]_h \rightarrow [\ ]_h\ b$ (*send–out communication rules*).

  (d) $[\, a \,]_h \rightarrow b$ (*dissolution rules*).

  (e) $[\, a \,]_h \rightarrow [\, b \,]_h\ [\, c \,]_h$ (*division rules for elementary membranes*).

  (f) $[\, [\,]_{h_1}\ [\,]_{h_2}\,]_h \rightarrow [\, [\,]_{h_1}]_h\ [\, [\,]_{h_2}]_h$ (*division rules for non–elementary membranes*).

- The sets $\mathcal{NAM}^0, \mathcal{AM}^0(\alpha, \beta, \gamma, \delta)$, where:
  - $\alpha \in \{-d, +d\}$.
  - $\beta \in \{-n, +n\}$.
  - $\gamma \in \{-e, +e\}$.
  - $\delta \in \{-c, +c\}$.

A Păun's conjecture

At the beginning of 2005, Gh. Păun (problem **F** from [1]) wrote:

> *My favorite question (related to complexity aspects in P systems with active membranes and with electrical charges) is that about the number of polarizations. Can the polarizations be completely avoided? The feeling is that this is not possible – and such a result would be rather sound: passing from no polarization to two polarizations amounts to passing from non–efficiency to efficiency.*

The so–called Păun's conjecture can be formally formulated:

$$\mathbf{P} = \mathbf{PMC}_{\mathcal{AM}^0(+d,-n,+e,+c)}$$

[1] Gh. Păun: Further twenty six open problems in membrane computing. *Third Brainstorming Week on Membrane Computing* (M.A. Gutiérrez et al. eds.), Fénix Editora, Sevilla, 2005, pp. 249–262.

Partial answers

- Limitations of $\mathcal{AM}^0$ which do not make use of dissolution rules:

  Theorem: $\mathbf{P} = \mathbf{PMC}_{\mathcal{AM}^0\,(-d,+n,+e,+c)}$ (Seville team, 2006).

  - The notion of dependency graph:
    - ⋆ Simulating accepting computations in $\mathcal{AM}^0\,(-d,+n,+e,+c)$ by means of reachability problems in a static directed graph.

- Efficiency of $\mathcal{AM}^0$ when dissolution and division for non–elementary membranes is permitted:
  - $\mathbf{PSPACE} \subseteq \mathbf{PMC}_{\mathcal{AM}^0\,(+d,+n,+e,+c)}$ (A. Alhazov, P-J, 2007).

- A borderline between efficiency and non–efficiency: dissolution rules in the framework of $\mathcal{AM}^0(+n,+e,+c)$.

Tissue-like membrane systems

- $\Pi = (\Gamma, \Sigma, \mathcal{E}, \mathcal{M}_1, \dots, \mathcal{M}_q, \mathcal{R}, i_{in}, i_{out})$
- Basic tissue P systems:
  - $(i, u/v, j)$, for $i, j \in \{0, 1, \dots, q\}, i \neq j$, and $u, v \in \Gamma^*$ (*symport-antiport rules*).
  - *Length* of the rule $(i, u/v, j)$: $|u| + |v|$
- Tissue P systems with cell division:
  - *Symport-antiport rules*.
  - $[\ a\ ]_i \to [\ b\ ]_i [\ c\ ]_i$, where $i \in \{1, 2, \dots, q\}$ and $a, b, c \in \Gamma$ (*division rules*).
- Tissue P systems with cell separation:
  - *Symport-antiport rules*.
  - $[\ a\ ]_i \to [\ \Gamma_1\ ]_i\ [\ \Gamma_2\ ]_i$, where $i \in \{1, 2, \dots, q\}$, $a \in \Gamma$, $i \neq i_{out}$ and $\{\Gamma_1, \Gamma_2\}$ is a fixed partition of Γ (*separation rules*).
- The sets $\mathcal{TC}$, $\mathcal{TDC}$, $\mathcal{TSC}$, and $\mathcal{TDC}(k)$, $\mathcal{TSC}(k)$, for each $k \geq 1$.

On efficiency of tissue P systems

- $\mathbf{P} = \mathbf{PMC}_{\mathcal{TC}}$ (Seville team, 2009).
- $\mathbf{P} = \mathbf{PMC}_{\mathcal{TDC}(1)}$ (Seville team, 2010).
- $\mathbf{P} = \mathbf{PMC}_{\mathcal{TSC}(2)}$ (L. Pan, P-J, A. Riscos, M. Rius, 2012).
- $\mathbf{NP} \cup \mathbf{co} - \mathbf{NP} \subseteq \mathbf{PMC}_{\mathcal{TDC}(2)}$ (A. Porreca, N. Murphy, P-J, 2012).
- $\mathbf{NP} \cup \mathbf{co} - \mathbf{NP} \subseteq \mathbf{PMC}_{\mathcal{TSC}(3)}$ (P. Sosík, P-J, 2012).
- Borderlines between efficiency and non–efficiency:
  - division rules in the framework of $\mathcal{TC}$.
  - length of communication rules in the framework of $\mathcal{TD}$: passing from 1 to 2 amounts to passing from non–efficiency to efficiency.
  - length of communication rules in the framework of $\mathcal{TS}$: passing from 2 to 3 amounts to passing fromm non–efficiency to efficiency.

Tissue P systems without environment

- Tissue-like P systems: $\Pi = (\Gamma, \Sigma, \mathcal{E}, \mathcal{M}_1, \dots, \mathcal{M}_q, \mathcal{R}, i_{in}, i_{out})$
  - The objects of $\mathcal{E}$ initially appear located in the environment in an arbitrary number of copies.
- Tissue-like P systems without environment: $\mathcal{E} = \emptyset$.
- The classes $\widehat{\mathcal{TC}}$, $\widehat{\mathcal{TDC}}$, $\widehat{\mathcal{TSC}}$, and $\widehat{\mathcal{TC}(k)}$, $\widehat{\mathcal{TDC}(k)}$, $\widehat{\mathcal{TSC}(k)}$, for each $k \geq 1$.

On efficiency of tissue P systems without environment

Division rules

- For each k: $\mathbf{PMC}_{\widehat{\mathcal{TDC}}(k+1)} = \mathbf{PMC}_{\mathcal{TDC}(k+1)}$ (Seville team, 2012).
  - $\mathbf{P} = \mathbf{PMC}_{\widehat{\mathcal{TDC}}(1)}$.
  - $\mathbf{NP} \cup \mathbf{co-NP} \subseteq \mathbf{PMC}_{\widehat{\mathcal{TDC}}(2)}$.
- The length of communication rules provides a new borderline of the efficiency in the framework $\widehat{\mathcal{TD}}$.

Separation rules

- $\mathbf{P} = \mathbf{PMC}_{\widehat{\mathcal{TSC}}}$ (Seville team, 2013).
  - $\mathbf{P} = \mathbf{PMC}_{\widehat{\mathcal{TSC}}(3)}$.
  - $\mathbf{NP} \cup \mathbf{co-NP} \subseteq \mathbf{PMC}_{\mathcal{TSC}(3)}$.
- The environment provides a new borderline of the efficiency in the framework $\mathcal{TSC}(3)$.

Conclusions: Frontiers of the efficiency

- Kind of the rules:
  - Division rules in $\mathcal{AM}(-n)$.
  - Division rules in $\mathcal{TC}$.
  - Dissolution rules in $\mathcal{AM}^0(+n, +e, +c)$.
- The length of communication rules:
  - Passing from 1 to 2 in $\mathcal{TD}$.
  - Passing from 1 to 2 in $\widehat{\mathcal{TD}}$.
  - Passing from 2 to 3 in $\mathcal{TS}$.
- The environment:
  - In the framework $\mathcal{TSC}(3)$.

Each of them provides a new way to attack the **P** versus **NP** problem.

THANK YOU!